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10ME64

Sixth Semester B.E. Degree Examination, Dec.2017/Jan.2018

Finite Element Methods

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

PART - A

- Describe the basic steps in the finite element method for engineering analysis in detail. (06 Marks)
 - For a 3-D elemental cube, obtain the differential equations of equilibrium subjected to a system of stresses in all the three directions. (06 Marks)
 - Distinguish clearly between plane stress and plane strain problems. Also give the constitutive equations (stress-strain equations) for both. (08 Marks)
- A cantilever beam of span 'l' is subjected to a uniformly distributed load P_0 over its entire length. The Young's modulus of elasticity of the beam material is 'E' and moment of inertia of the section is 'I'. Derive an equation for deflection by using the Rayleigh-Ritz method. (12 Marks)
 - Derive the element stiffness matrix for a two-node one-dimensional bar element using direct approach. (08 Marks)
- Sketch and explain Pascal triangle for 2-D polynomials. (04 Marks)
 - Derive the strain displacement matrix [B] for a three-noded constant strain triangle (CST) element. (08 Marks)
 - Derive the Jacobian matrix [J] for a four-noded quadrilateral element. (08 Marks)
- Explain in detail, 'Elimination approach' to handle boundary conditions. (10 Marks)
 - For the three stepped bar shown in Fig.Q4(b), find the nodal displacements, stress in the middle portion and left support reaction. (10 Marks)

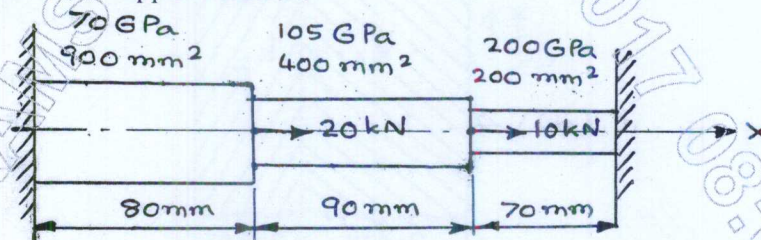


Fig.Q4(b)

(10 Marks)

PART - B

- Derive the shape functions for a four node 1-D cubic bar element and show the variations of them along the length of the element. (10 Marks)
 - Using two point Gaussian quadrature formula evaluate the following integrals:

$$i) \int_{-1}^{+1} \int_{-1}^{+1} (r^2 + 2rs + s^2) dr ds$$

$$ii) \int_0^1 x^2 dx$$

(10 Marks)

- 6 a. Derive the element stiffness matrix for the truss element. (08 Marks)
 b. For the two-bar truss shown in Fig.Q6(b), determine the nodal displacement, stress in each element and reaction at the support.
 Take $E = 2 \times 10^5 \text{ N/mm}^2$, area of each bar = $A_e = 200 \text{ mm}^2$.

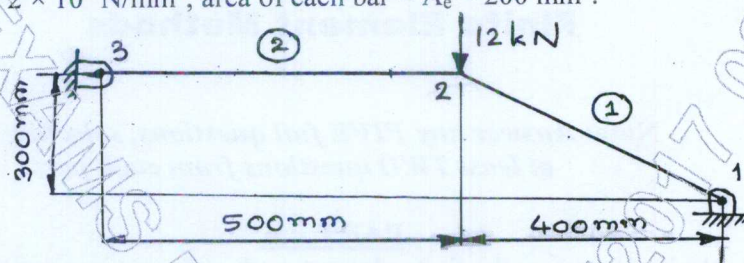


Fig.Q6(b)

(12 Marks)

- 7 a. Obtain the shape functions for a 2-node beam element and plot them. (10 Marks)
 b. Determine the maximum deflection of the cantilever beam with uniform cross section as shown in Fig.Q7(b), by assuming the beam as a single element. Take $E = 7 \times 10^9 \text{ N/m}^2$, $I = 4 \times 10^4 \text{ m}^4$.

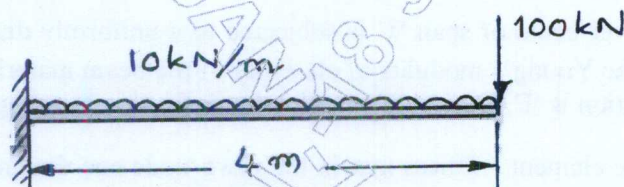


Fig.Q7(b)

(10 Marks)

- 8 a. Explain the different types of boundary conditions in heat transfer problems. (08 Marks)
 b. A composite wall consists of two materials is as shown in Fig.Q8(b). The outer temperature is $T_0 = 20^\circ\text{C}$. Convection heat transfer takes place on the inner surface of the wall with $T_\infty = 800^\circ\text{C}$ and $h = 25 \text{ W/m}^2\text{-}^\circ\text{C}$. Determine the temperature distribution in the wall.

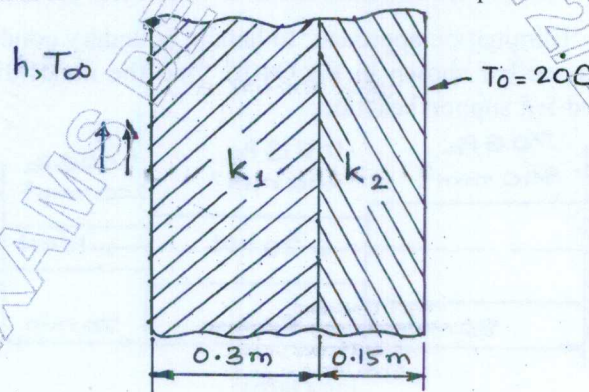


Fig.Q8(b)

$$\begin{aligned} k_1 &= 20 \text{ W/m-}^\circ\text{C} \\ k_2 &= 30 \text{ W/m-}^\circ\text{C} \\ h &= 25 \text{ W/m}^2\text{-}^\circ\text{C} \\ T_\infty &= 800^\circ\text{C} \end{aligned}$$

(12 Marks)
